Enforcing Topological Constraints in Random Field Image Segmentation

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1 Introduction

- Random field image segmentation:
  find the labeling of pixels, $y : \mathcal{V} \to \{0,1\}$, minimizing the *RF energy*

$$E(y;\mu,\eta,\omega) = \sum_{v_i \in \mathcal{V}} \mu_i y_i + \omega \sum_{(v_i,v_j)\in\mathcal{E}} \eta_{ij} \cdot |y_i - y_j|$$

- Solution: Graph-Cut (GC) [Boykov et al. PAMI01] .
- Issue: tend to cut thin trunks and disconnect the object.

Old Problem. *minimizing the RF energy under topological constraints*

$$\operatorname{argmin}_y E(y;\mu,\eta,\omega), \text{ s.t. } FG(y) \text{ is connected.}$$

- Previous works
  - NP-hard (unary-only), interactive input [Vicente et al. CVPR08]
  - Local minimum [Zeng et al. CVIU08]
  - Linear programming (CMRF) [Nowozin and Lampert CVPR09]

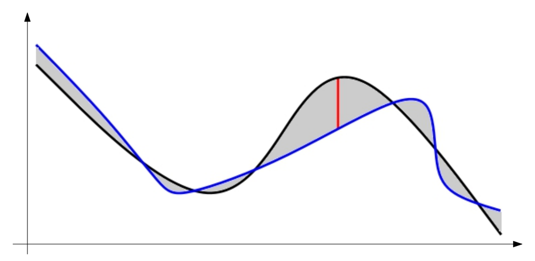
2 Our Contributions

New Problem. *find the closest perturbation of the unary term, s.t. the optimal solution is connected.*

$$\operatorname{argmin}_{\widehat{\mu}} \|\widehat{\mu} - \mu\|_p, \text{ s.t. } FG(\operatorname{argmin}_y E(y;\widehat{\mu},\eta,\omega)) \text{ is connected .}$$

How close is a perturbation, $\widehat{\mu} : \mathcal{V} \to \mathbb{R}$

- $\|\widehat{\mu} - \mu\|_p = \sqrt[p]{\sum_{v_i\in\mathcal{V}} |\widehat{\mu}_i - \mu_i|^p}$
- $\|\widehat{\mu} - \mu\|_\infty = \max_{v_i\in\mathcal{V}} |\widehat{\mu}_i - \mu_i|$

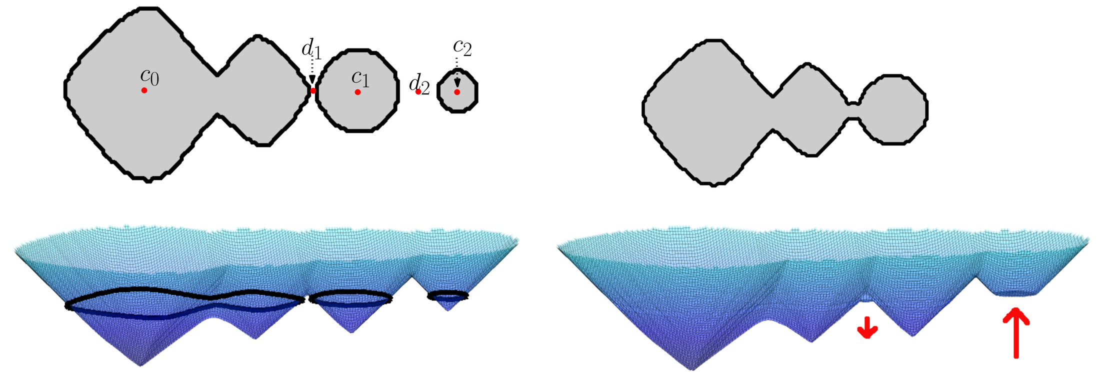


Results

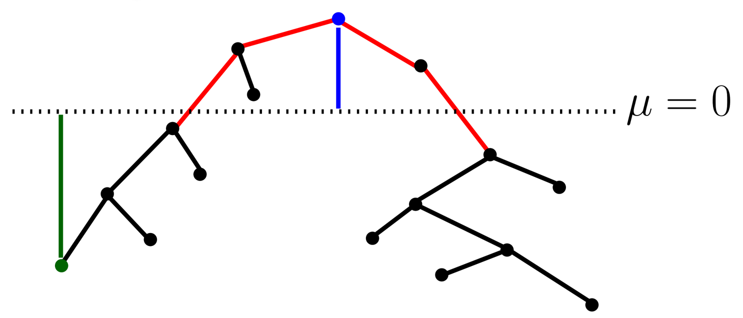
- Unary-Only ($\omega = 0$)
  - $p = 1$, New Problem = Old Problem; $p < \infty$, New Problem is NP-hard
  - $p = \infty$, a polynomial algorithm ($O(n \log n)$)
- With binary ($\omega > 0$):
  A heuristic algorithm. Works well in practice (good results, efficient)

2.1 Unary-only, $p = \infty$

- The optimal solution is $y_i^* = 1$ iff $\mu_i \le 0$, and 0 otherwise.
- Eliminating individual components (basins) by perturbing the terrain, choose the best of the two options.
  - Opt. 1: remove the whole basin, cost=depth of the basin
  - Opt. 2: merge to a deeper one, by digging a canal with the lowest cost
    cost=the maximal height along the canal path



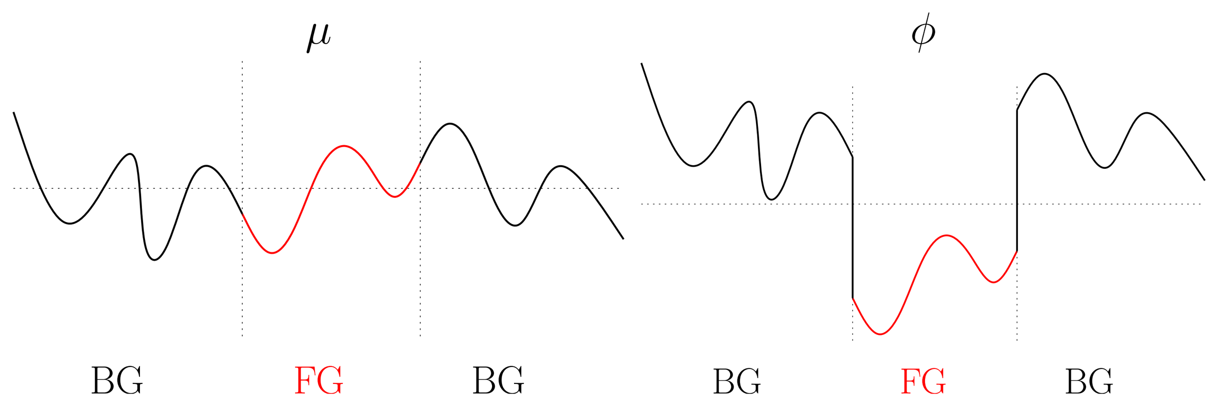
- Eliminating all components:
  - Find the optimal way for each component efficiently
  - Theorem [Bendich et al. ESA10]: For every component, there is a *critical point* deciding the cost of eliminating it.
  - Algorithm: build component merging tree, eliminate all subtrees below $\mu = 0$



2.2 With binary term, $\omega > 0$

To use unary-only algorithm, need ϕ, so that $y_i^* = 1$ iff $\phi_i \le 0$.

$$\phi_i = \begin{cases} \mu_i + C \text{ if } y_i = 0 \\ \mu_i - C \text{ if } y_i = 1 \end{cases} \text{ where } C = \max|\mu_i|$$



Algorithm (When $\omega = 0$, the same as the unary-only algorithm):

- $\widehat{\mu} = \mu$, $y^* = \operatorname{argmin}_y E(y;\widehat{\mu},\eta,\omega)$
- While $FG(y^*)$ NOT connected do
  - Create ϕ using $\widehat{\mu}$ and y^*
  - Compute $\widehat{\phi}$ using unary-only algorithm
  - $\widehat{\mu} = \widehat{\mu} + \widehat{\phi} - \phi$
  - $y^* = \operatorname{argmin}_y E(y;\widehat{\mu},\eta,\omega)$

3 Experiments

3.1 Natural Images (Compared With GC)

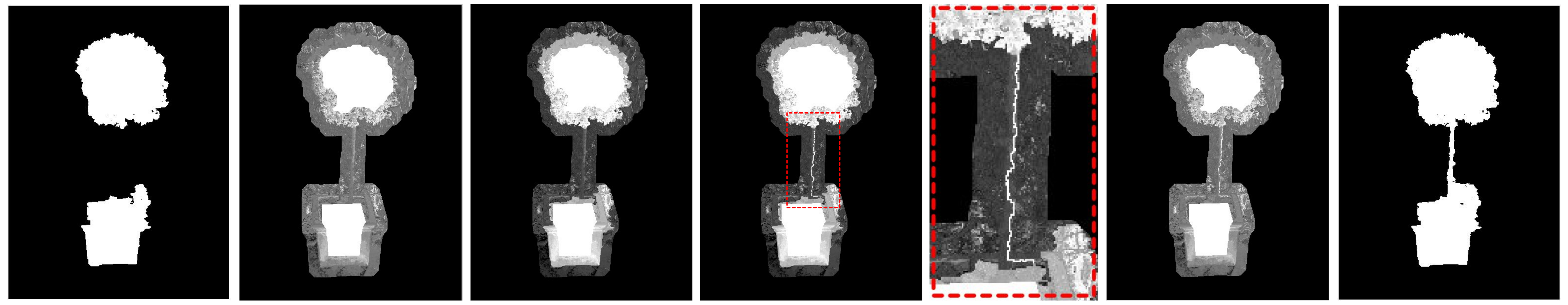
- Details:
  - Unary: Gaussian mixture model
  - Binary [Blake et al. ECCV2004]: $\eta_{i,j} = \exp(-\frac{\|Color_i - Color_j\|^2}{2\sigma^2})$
  - Running Time: ≤ 3 GC time
  - GrabCut Database (50 images), Trimap, $\omega \in \{0.0, 0.2, \cdots, 1.4\}$
- Conclusion:
  - when GC result not connected, BETTER
  - when GC result connected, the same
  - improves the numerical results (slightly)

method	err (global w) [%]	err (per-image w) [%]
TopoCuts	7.0 ± 0.7	6.1 ± 0.6
GC	7.5 ± 0.7	6.5 ± 0.6

3.2 Artificial Images (Compared with GC, GC-Max-Connected-Component, CMRF)

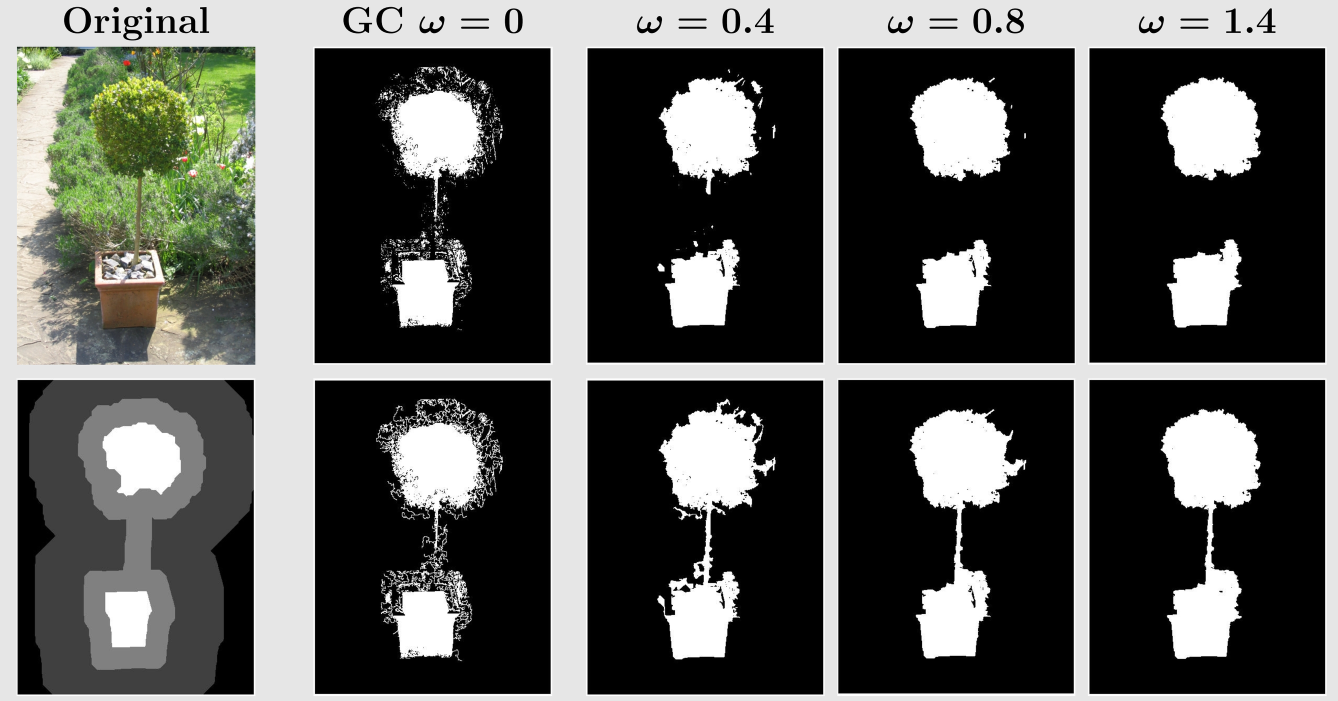
When noise is correlated (please see the paper for details):

- Better than GC and GCMCC
- Same as CMRF, much more efficient

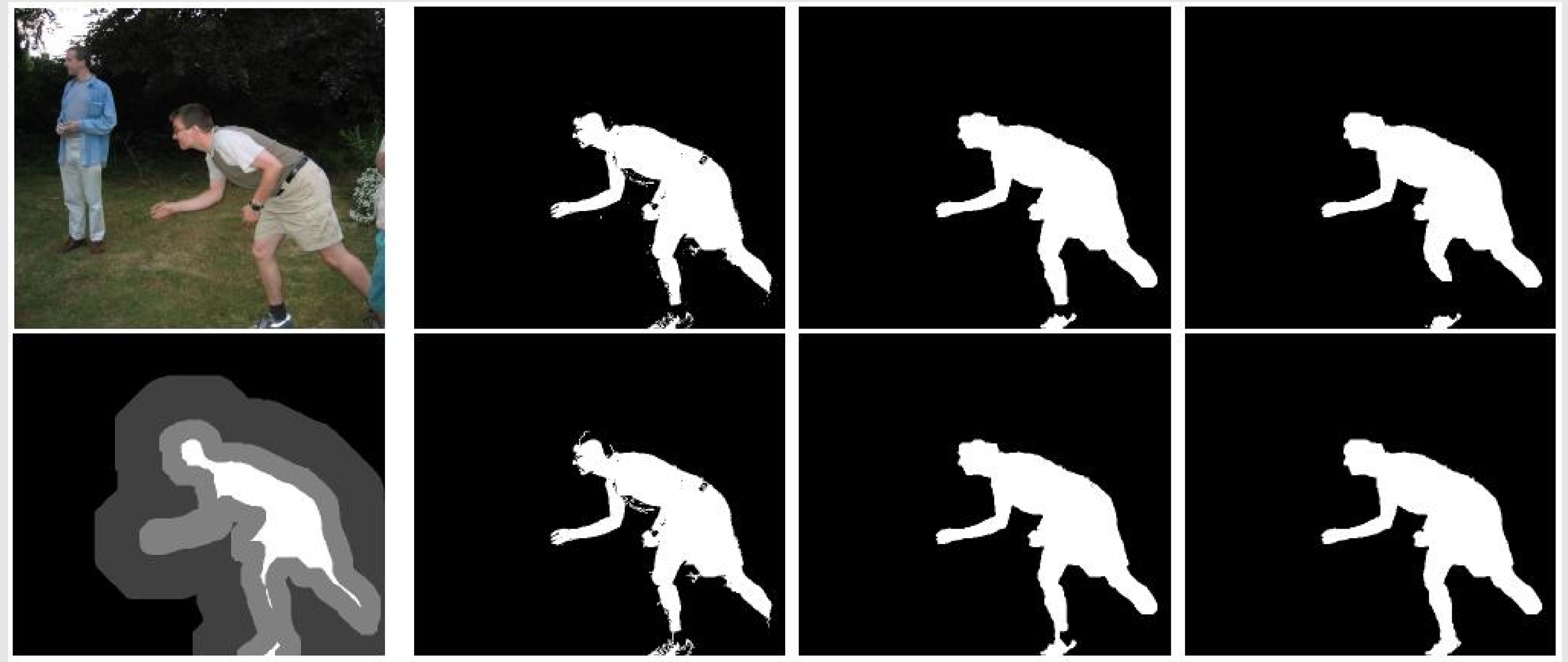


GC Seg → Original μ → Original ϕ → Adjusted ϕ Zoom in → Adjusted μ → Fixed Seg

Original GC $\omega = 0$ $\omega = 0.4$ $\omega = 0.8$ $\omega = 1.4$



Input (TriMap) TopoCut $\omega = 0$ $\omega = 0.4$ $\omega = 0.8$ $\omega = 1.4$



$\omega = 0.0, 0.4, 0.8$

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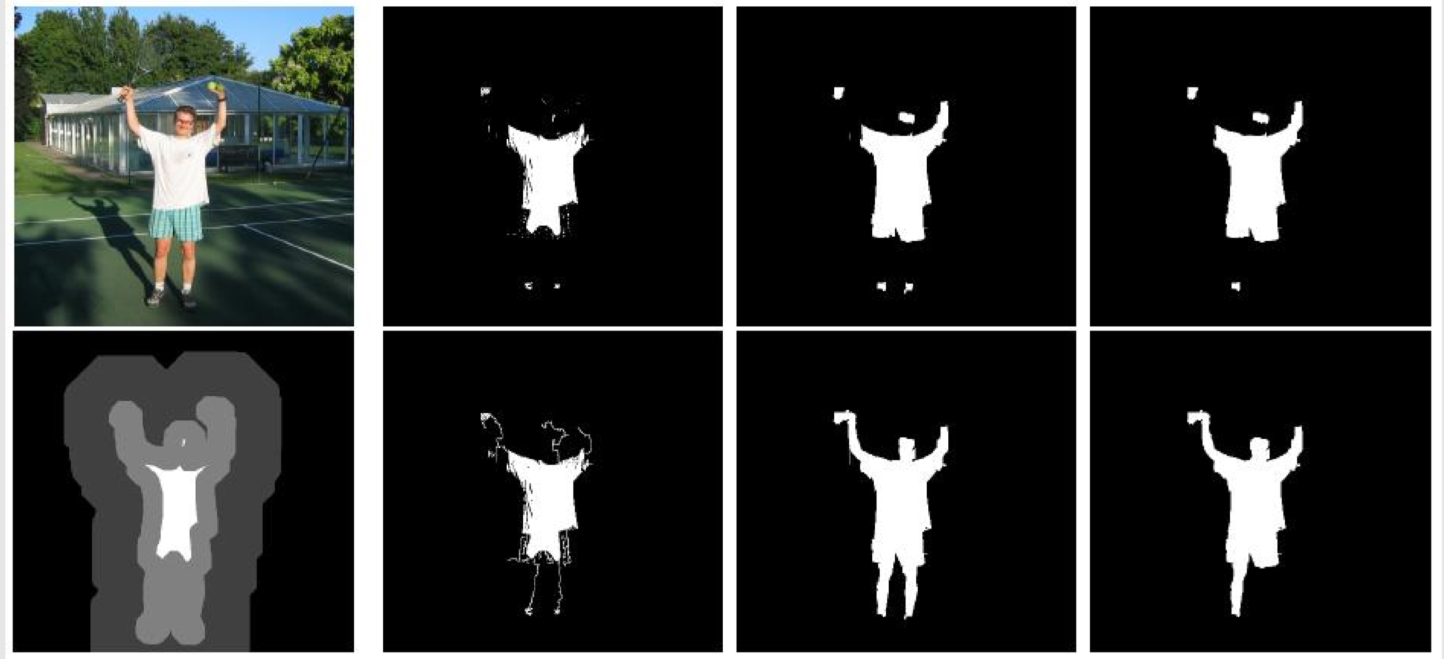

$\omega = 0.0, 0.4, 0.8$