Diffusion Runs Low on Persistence Fast

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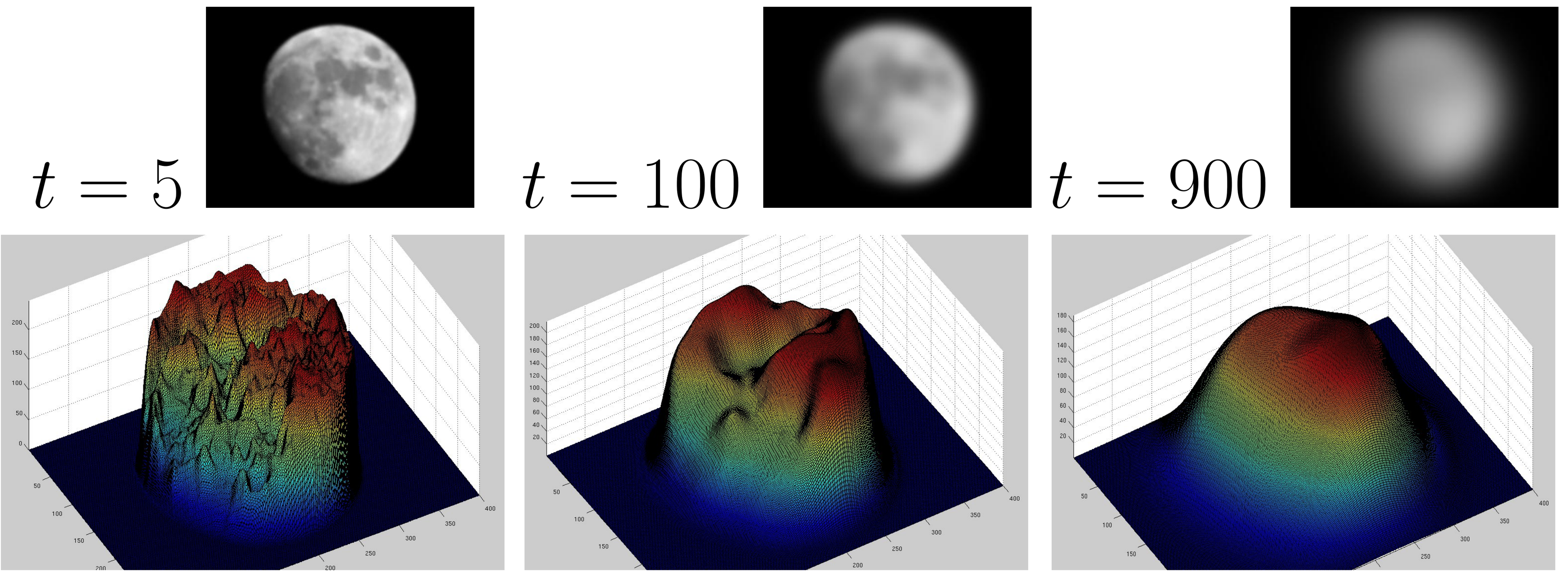
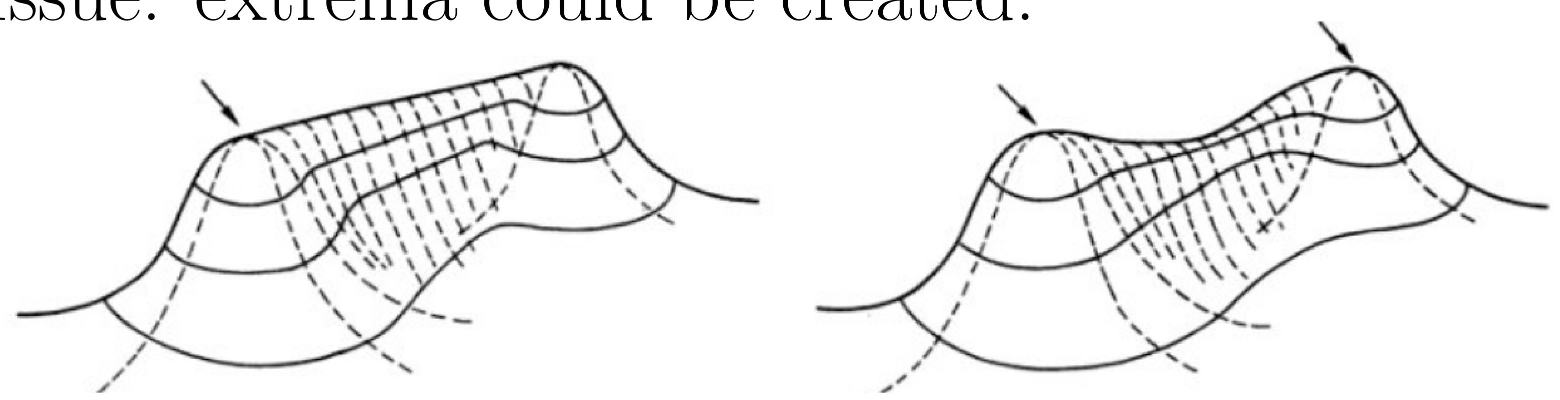
Objective. *Quantitatively prove the scale-space intuition: Extrema are quickly smoothed out as scale increases.*

- Scale-space: $\{I_t = I * g_t \mid t \in (0, \infty)\}$, Scale: t
- Gaussian kernel: $g_t(x) = \frac{1}{2\pi t} exp\left(-\frac{\|x\|^2}{2t}\right)$



- Issue: extrema could be created.



Picture from Linderberg Thesis 1991

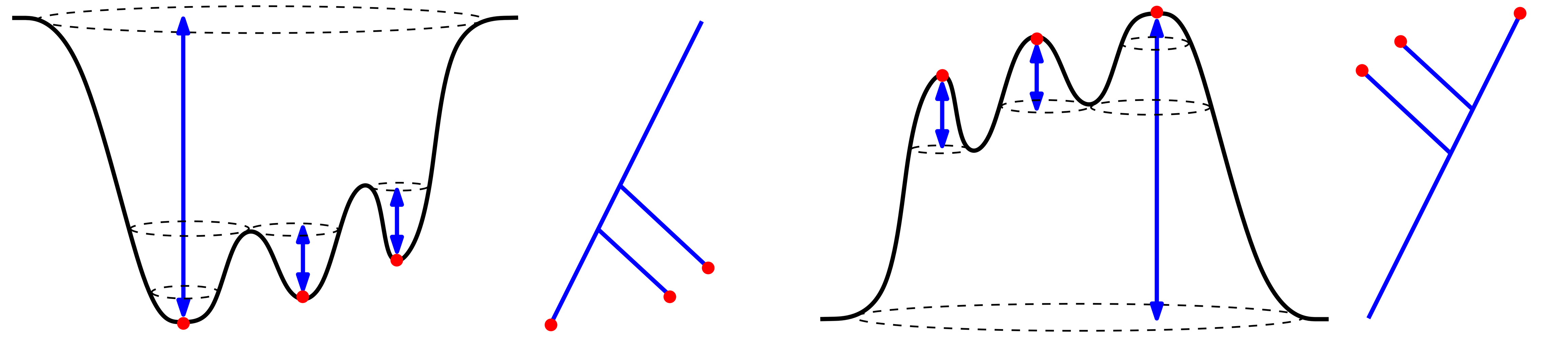
- Linderberg PAMI 93:
for random noise images, the expected number of extrema in I_t is $const/t$.
- Our contribution:

Main Theorem. *Weigh extrema by their* ***persistence****.*
The total weight (p-norm) is no more than const/t.

$$\left(\sum_{x \text{ extrema of } I_t} pers(x)^p \right)^{1/p} \leq const/t$$

What is persistence?

- Definition:
  - $f : \Omega \to \mathbb{R}$, grow the sublevel set $\{x \in \Omega | f(x) \leq \theta\}$ as θ increases from $-\infty$ to $+\infty$.
  - Topological features (components, holes) are created and destroyed.
  - Minimum: persistence is the life length of the component it creates.
  - Maximum: persistence is the life length of the hole it destroys.



- Computation:
  - Discretize, boundary matrices, reduction.
  - For 1024×768 image, takes 4 seconds.
  - Code available at `http://pub.ist.ac.at/~cchen`